1. PHYSICAL MOTIVATION

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- BOSE, EINSTEIN (1924): Theoretical predictions

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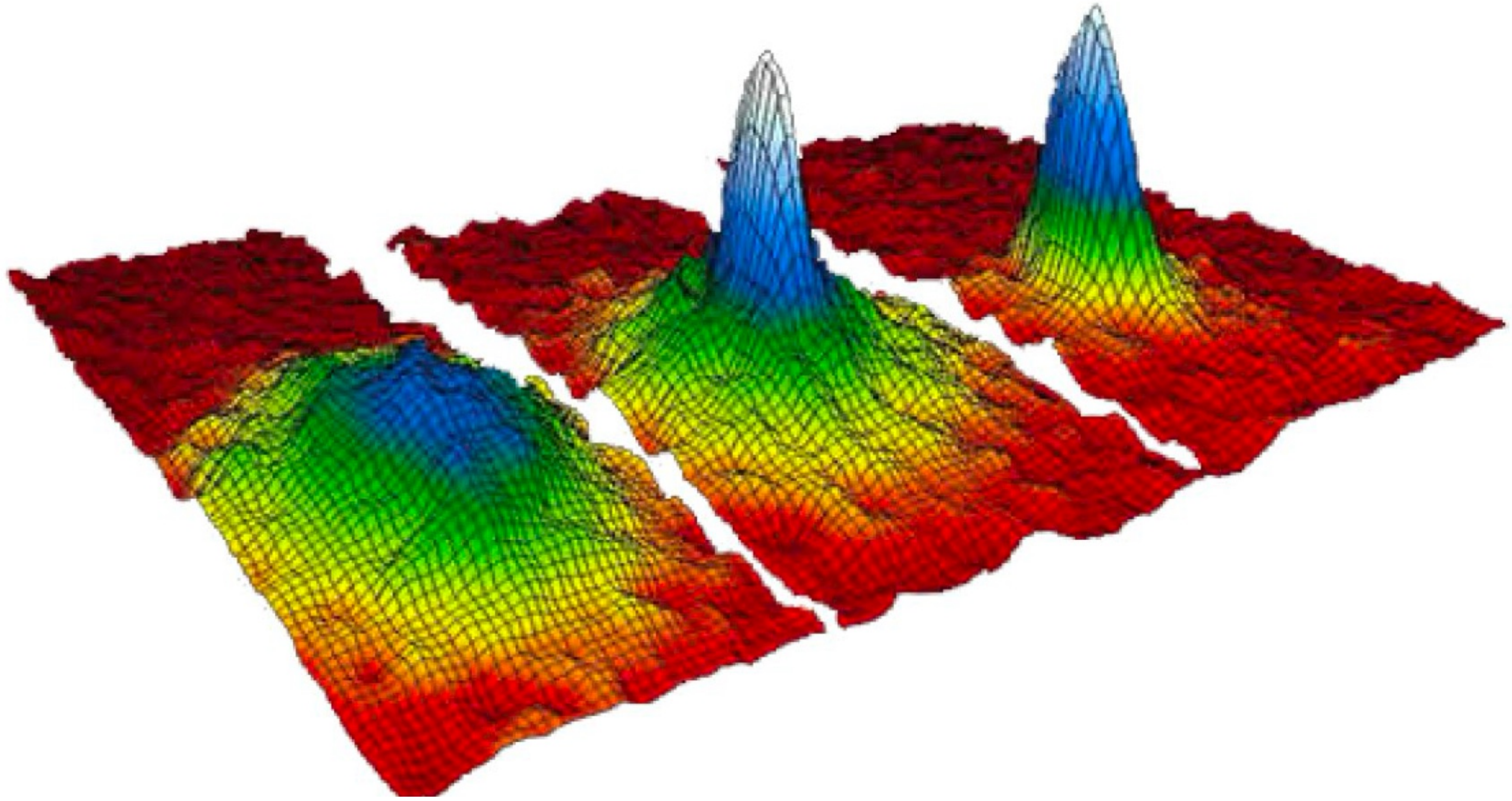

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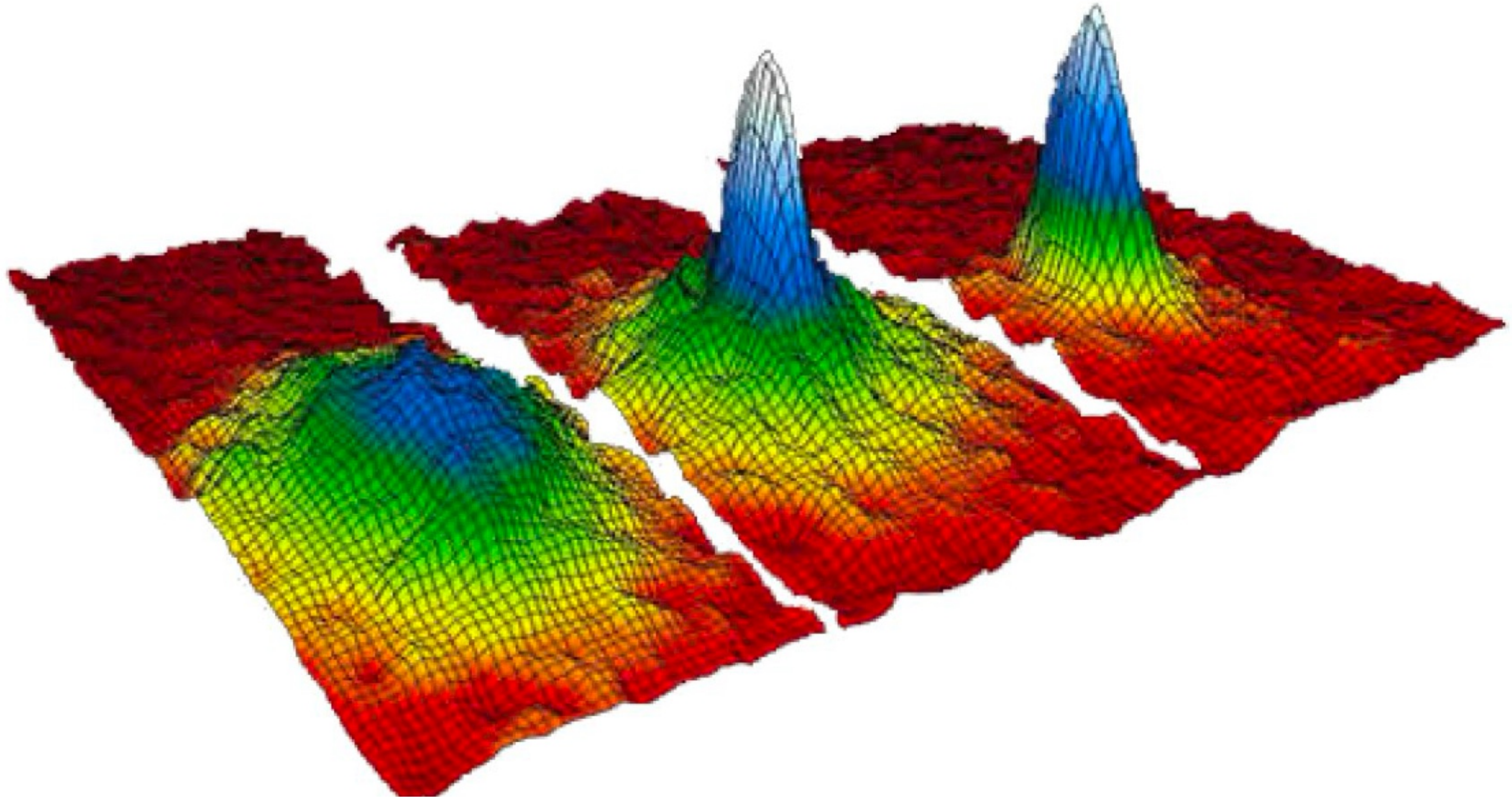

$\boxed{\text{GOAL:}}$

MICROSCOPIC DESCRIPTION

N- body system

$\Psi_N \in L^2_s(\mathbb{R}^{3N})$

$N \to \infty$

MACROSCOPIC DESCRIPTION

ONE - PARTICLE SYSTEM

$\varphi \in L^2(\mathbb{R}^3)$

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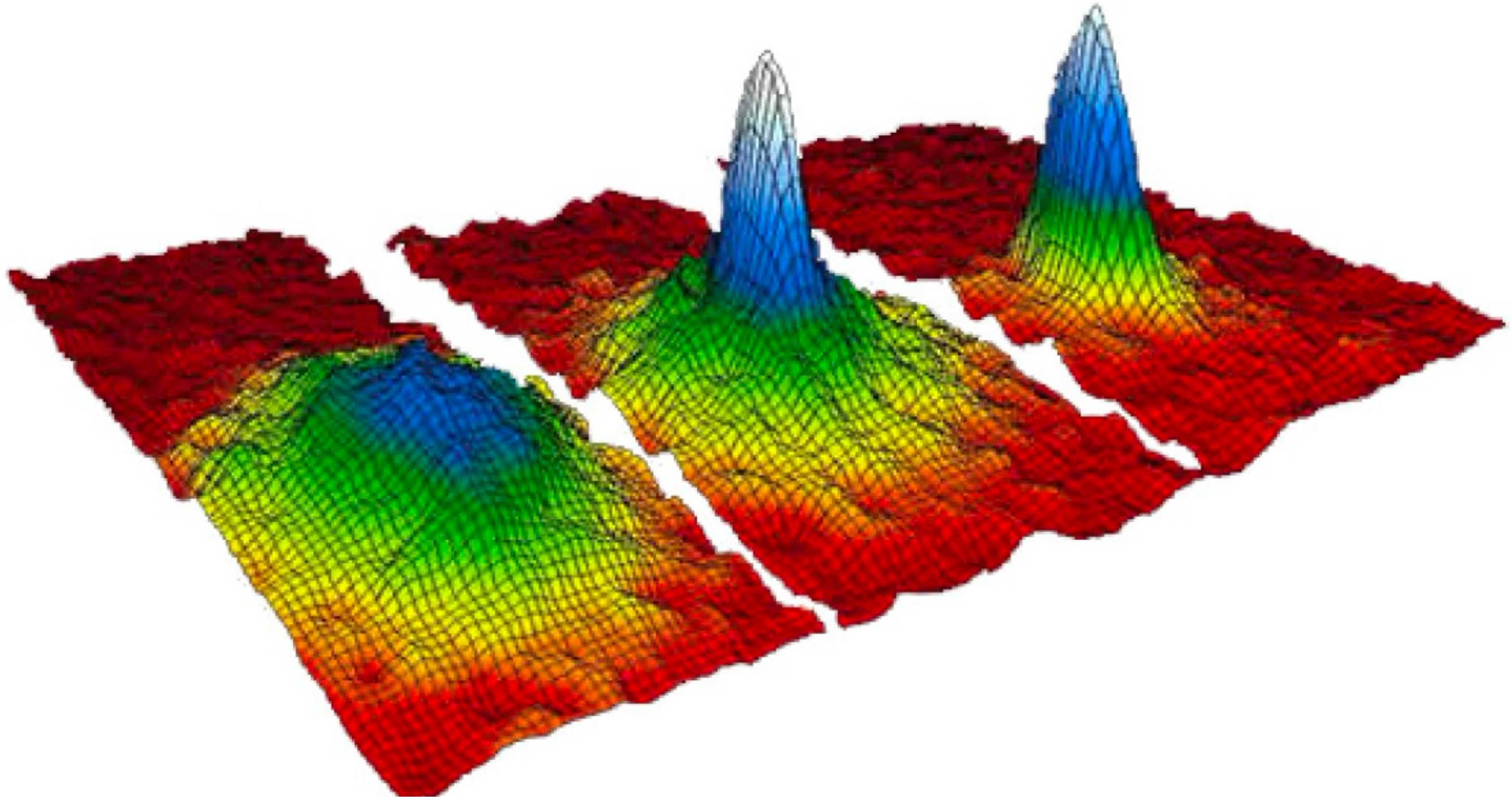

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MICROSCOPIC DESCRIPTION $\xrightarrow{N \to \infty}$ MACROSCOPIC DESCRIPTION

N - body system $\longrightarrow$ ONE - PARTICLE SYSTEM

$\Psi_N \in L^2_s(\mathbb{R}^{3N})$ $\qquad$ $\varphi \in L^2(\mathbb{R}^3)$

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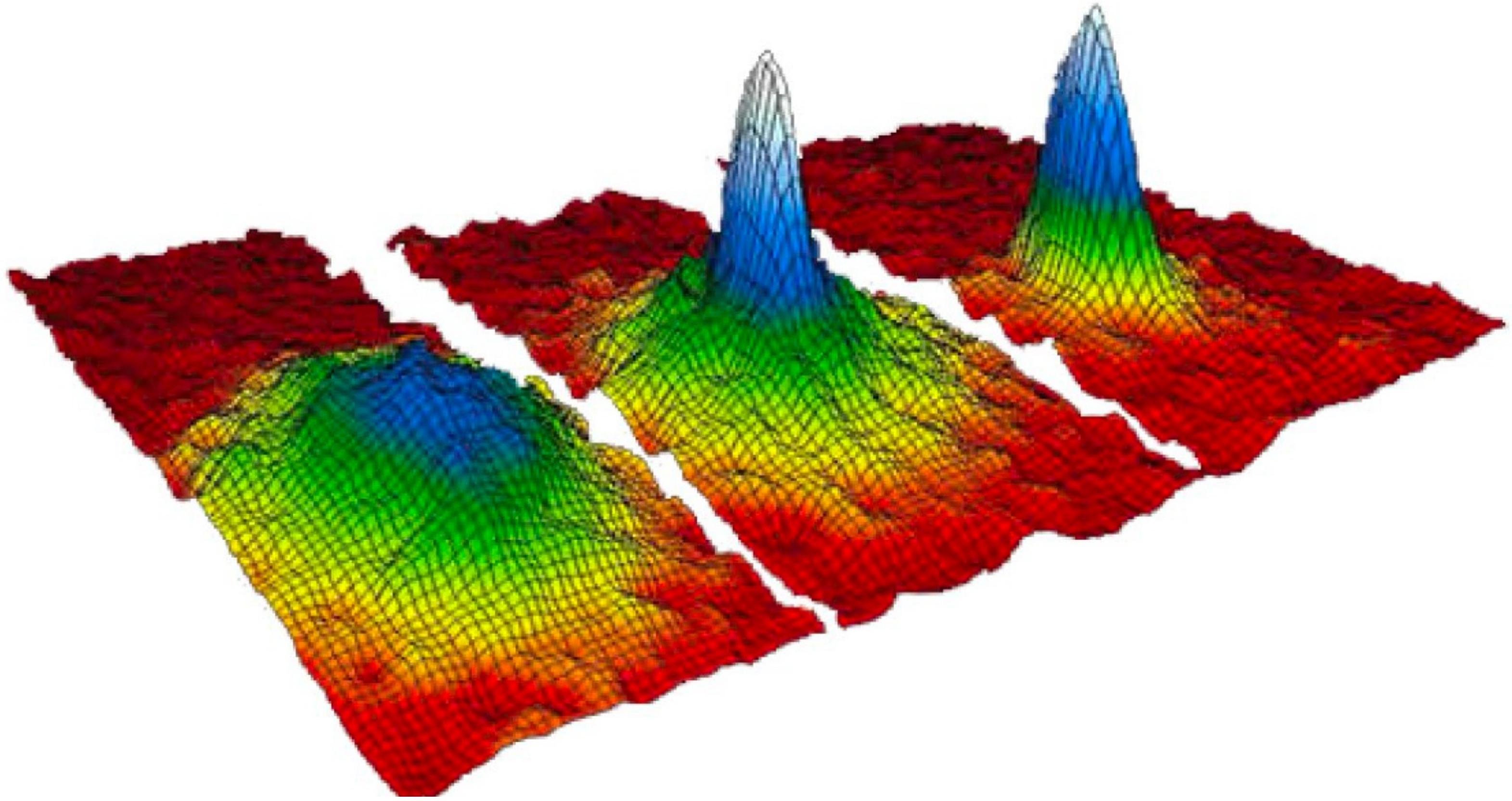

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LLN

CLT

LDE

2. MATHEMATICAL DESCRIPTION

2. Mathematical Description

2.1. Wave Functions

- N- particle quantum system described by

$$\Psi_N \in L^2(\mathbb{R}^{3N}), \quad \Psi_N : \underbrace{(x_1, \dots, x_N)}_{\text{position of particles}} \longrightarrow \mathbb{C} \quad , \text{ s.t. } \|\Psi\|_2 = 1 .$$

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Ex $\Psi_N = \varphi^{\otimes N}, \quad \varphi \in L^2(\mathbb{R}^3)$

called factorized state

2.2 OBSERVABLES

- associated to self-adjoint operators O
- Measurement of O

$$\langle \Psi_N, O \Psi_N \rangle = \int \lambda \, \langle \Psi_N, dE_O(\lambda) \Psi_N \rangle$$

spectral thm: $O = \int \lambda \, dE_O(\lambda)$

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$$O = \sum_j \lambda_j |\varphi_j\rangle\langle\varphi_j|$$

eigenvalues

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eigenvalues (λ_j), eigenfcts. (φ_j)

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$$\langle \Psi_N, p_j \Psi_N\rangle = \int p_j \, |\hat{\Psi}_N(p_1, \dots p_N)|^2 \, dp_1 \cdots dp_N$$

p_j " $-i\nabla_j$

probability density

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probability density

DEF: For self-adjoint one-particle operator $O^{(1)}$,

$$O_i^{(1)} = 1 \otimes \cdots \otimes 1 \otimes \overset{i}{O^{(1)}} \otimes 1 \otimes \cdots 1$$

and $\Psi_N \in L_s^2(\mathbb{R}^{3N})$, let Y_i^N denote random variable with

$$\mathbb{P}_{\Psi_N}[Y_i^N \in A] = \langle \Psi_N, \mathbb{1}_A(O_i^{(1)}) \Psi_N \rangle \qquad \text{for } A \subset \mathbb{R}.$$

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$O^{\cancel{(1)}\,(k)}_{\cancel{i}\; i_1,\dots,i_k}$

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Ex Factorized state

Z^N_i associated to $\varphi^{\otimes N}$

⇝ blackboard

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Ex Factorized state

Z^N_i associated to $\varphi^{\otimes N}$ correspond to independent, identically distributed random variables.

2.3 REDUCED DENSITIES

DEF: For $\Psi_N \in L^2_s(\mathbb{R}^{3N})$, let

$$\gamma^{(1)}_{\Psi_N} := N \, \mathrm{Tr}_{2,\dots,N} \underbrace{|\Psi_N\rangle\langle\Psi_N|}_{\text{orthogonal projection on } \Psi_N}$$

denote the one-particle reduced density with

$$\gamma^{(1)}_{\Psi_N}(x,y) = N \int dx_2 \cdots dx_N \; \Psi_N(x, x_2, \dots, x_N) \overline{\Psi_N}(y, x_2, \dots, x_N)$$

and $\mathrm{tr}\, \gamma^{(1)}_{\Psi_N} = N$.

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and $\operatorname{tr}\gamma^{(\cancel{1})k}_{\Psi_N} = \binom{N}{k}$

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RMK: γ^{i}_{N} ass. to $\Psi_N \in L^2_s(\mathbb{R}^{3N})$ $\leadsto$ blackboard

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RMK: Y^i_N ass. to $\Psi_N \in L^2_s(\mathbb{R}^{3N})$ are identically distributed.

2.4 DYNAMICS

- **SCHRÖDINGER EQ:** Let $\Psi_{N,t} \in L^2_s(\mathbb{R}^{3N})$ solve

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$$H_N = \overbrace{\sum_{i=1}^{N} \left(-\Delta_{x_i}\right)}^{\text{kinetic energy}} + \underbrace{\lambda \sum_{1\le i<j\le N} v(x_i - x_j)}_{\text{interaction energy}}$$

coupling constant

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- **INITIAL STATES**: $\Psi_{N,0} = \varphi_0^{\otimes N}$ BUT: $\Psi_{N,t} \neq \varphi_t^{\otimes N}$

- **HOWEVER**: $\Psi_{N,t}(x_1, \dots, x_N) \approx \varphi_t(x_j)^{\otimes N}$

with $$i\partial_t \varphi_t = (-\Delta + v * |\varphi_t|^2)\varphi_t$$

Hartree eq.

and $\varphi_0 \in H^1(\mathbb{R}^3)$, $\|\varphi_0\| = 1$.

GOAL :

▶ **DEF** : Random variables $\{Y_i^{N,t}\}_{1 \le i \le N}$ associated to $\mathcal{Y}_{N,t}$.

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RMK : blackboard

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GOAL : characterize through

- Law of large numbers
- central limit theorem
- Large deviations